CO2-EMISSION REDUCTION BY MEANS OF ENHANCED THERMAL CONVERSION EFFICIENCY OF ICE CYCLES ESPECIALLY FOR USE IN HYBRID VEHICLES

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ABSTRACT

Most automobile manufacturers have developed hybrid vehicles that combine an internal combustion engine and an electric motor, fusing the advantages of these two power sources. For example, Toyota, in its Prius II, uses a highly efficient gasoline engine based on a modified Atkinson cycle featuring a variable valve timing management. This implementation of the Atkinson cycle is not the optimal solution because some of the air is first sucked from the intake manifold into the cylinder and subsequently returned. This oscillating air stream considerably reduces the thermal conversion efficiency of this cycle.

This paper analyzes in detail the loss of thermal conversion efficiency of an internal combustion engine - especially for modified Seiliger and Atkinson cycles - and a proposal is made for the improvement of aspirated and supercharged engines.

Keywords: Thermal Efficiency, ICE Cycle, Hybrid Vehicle, CO2 Reduction

1. INTRODUCTION

Most automobile manufacturers have been actively developing various new technologies aimed at reducing fuel consumption and diversifying energy sources, which is necessitated by the dwindling supply of petroleum resources. For example, in motive power sources for automobiles alone, they have been continuously improving conventional engines and have developed and commercialized lean-burn gasoline engines, direct-injection gasoline engines and common rail direct-injection diesel engines, etc. They have also been modifying Internal Combustion Engines (ICE) so that these can use alternative fuels, such as compressed natural gas, instead of gasoline or light oil, and have been installing these engines in commercially available vehicles. Toyota, Honda etc., have also developed and marketed hybrid vehicles that combine an engine and an electric motor, merging the advantages of these two power sources.

As is well known [1], [2], [3] the thermal conversion efficiency of the ICE cycles increases when the effective compression ratio grows up and/or the effective expansion becomes completed.

In conventional engines, because the volumetric compression and expansion strokes are nearly identical and the cylinder is completely filled, the effective compression ratio and the effective expansion ratio are basically identical as shown on the left side of Figure 1 for the modified Seiliger cycle (an ideal model of engine cycles), where p0 is the ambient pressure.

In the classical Seiliger cycle [3] or limited pressure cycle [2] the heat is released by constant volume (V) and constant pressure (p). For this reason this cycle is here referred to as the V,p,-cycle. In the modified Seiliger cycle, presented in Figure 1, the heat is released by constant volume, constant pressure and constant temperature. For this reason, this cycle can be referred to as the V,p,T-cycle. In this way, it becomes possible to generate ideal engine cycles which provide a more accurate model of the real ICE cycles by reaching their mechanical and thermal limits.



Figure 1. Schematic Pressure-Volume diagrams of the classical four stroke Seiliger and Atkinson cycle. The heat release in both cycles is modelled by constant volume (V), pressure (p) and temperature (T).

In conventional engines, any attempt to increase the expansion ratio ϵ_e also increases the volumetric compression ratio ϵ_c (computed by means of the volumetric strokes), inevitably resulting in knocking by SI engines or in exceeding the maximum allowed pressure by diesel engines. These consequences set a limit on how much the expansion ratio can be increased and also, as result, on the thermal conversion efficiency η_{th} . On the other hand the attempt to enhance the supercharging pressure (approx. equal to p₁) while preserving the volumetric compression ratio leads to the same restrictions which limit as well the thermal conversion efficiency level.

During the last few years the market share of hybrid vehicles using Spark Ignition (SI) engines has steadily increased. For example Toyota [4] uses a SI engine in its Prius II, which tries to achieve a high level of efficiency by using a modified (i.e. a four stroke) Atkinson cycle as shown on the right side of Figure 1 (classical Atkinson cycle has only two strokes). Because the expansion ratio is increased in comparison to the

effective compression ratio and the latter is at a high level, this engine, using this cycle, should theoretically yield a higher thermal conversion efficiency.

Consequently in the initial stage of the compression stroke (when the piston begins to ascend), some of the air that has entered the cylinder is returned to the intake manifold, in effect delaying the start of the effective compression [4]. In this way, the expansion ratio is increased without increasing the effective compression ratio. Since this method can increase the throttle valve opening time, it can reduce the negative pressure in the intake pipe during partial load, thus reducing intake losses. Sophisticated variable valve timing must be used to carefully adjust the intake valve timing to operating conditions in order to obtain maximum efficiency.

This implementation of the modified Atkinson cycle is not the optimal solution because e.g. some of the air is first sucked from the intake manifold into the cylinder and subsequently returned. Consequently the oscillating air stream considerably reduces the thermal conversion efficiency of the cycle [5].

The implementation of the Atkinson cycle especially for supercharged engines is analyzed in detail in the following sections of this paper and proposals for improvement are made.

2. WAYS FOR IMPROVING THE THERMAL CONVERSION EFFICIENCY OF ICE CYCLES

ASPIRATED ENGINES

In the case of an aspirated engine in which the intake valve is kept open for a large proportion of the compression process and the volumetric compression ratio is enhanced (like Toyota have done in the Prius II), the advantage of performing the cycle using the Atkinson principle is of little benefit for the following reasons (see [5] for more details):

• The oscillating air stream to and from the intake manifold, through the intake valve port, reduces considerably the thermal conversion efficiency of the cycle.

• The indicated fuel conversion efficiency gain is modest and is largely dependent on the fine tuning of all parameters (valve, timing etc.). On the contrary, such variable valve management is to complex and expensive for its modest efficiency gain.

• The specific power of the engine is too low because of the decreased retained mass of fresh change in cylinder before compression. This means that a relatively large (due to the large displacement) and therefore heavy engine is needed to power the vehicle.

In order to realize a strict Atkinson cycle - i.e. shortened compression and extended expansion - a special crankshaft drive is proposed, which allows geometrically different strokes for compression and expansion (see [5] and Figure 2). The design of this crankshaft drive is not the subject of this investigation and is therefore not described here. The oscillating air stream to and from the intake manifold, through

intake valve port, is, accordingly, eliminated and the thermal conversion efficiency of this cycle becomes as much as 15% higher [5] compared with a classical ICE cycle.

The analysis instruments used in [5] are based on a self developed model and simulation program for real ICE cycles, where the gas exchange processes are very accurately modeled. Such models and simulation programs need a lot of input data and have several influence parameters (see [6] and [7]).



Figure 2. Relative Piston Displacement – Crank Angle Diagram for a very high pressure supercharged engine

These are the reasons why the analysis of the influence parameters upon the thermal conversion efficiency is complicated and time expensive and, therefore, an optimal combination of these parameters is very difficult to obtain.

SUPERCHARGED ENGINES

In the case of supercharged ICE, the number of parameters which influence the thermal conversion efficiency becomes much higher. As a consequence, the effort to achieve combinations of parameters which maximize the thermal conversion efficiency of such real ICE cycle becomes much more difficult.

For these reasons, ideal models of ICE cycles of the modified V,p,T-Seiliger and -Atkinson cycles are developed here for this purpose. The simulation tool AVL BOOST® of AVL Co. is used as reference in order to evaluate their accuracy and the significance of their predictions upon the thermal conversion efficiency compared with the real models of ICE cycles.

Comparison of Seiliger and Atkinson Cycles using the ideal V,p,T - Model

The modeling by means of V,p,T–Seiliger and –Atkinson cycles has the advantage that it allows users to generate ideal engine cycles, which model the real ICE cycles more accurately as classical ideal V- and V,p-cycles by meeting their mechanical

(pressure) and thermal limits. A simple V-cycle (Otto cycle) where the heat is released only in an isochoric manner (constant volume) generates unrealistically high maximum pressure and temperature levels during the cycle. The attempt to limit the maximum pressure level leads to the classic V,p-cycle [1], [2], [3], where the heat is released in an isochoric and isobaric (constant pressure) manner. The V,p-cycles (i.e. classical Seiliger cycles) leads, for example, to very high temperature levels, in the case of fully loaded supercharged engines, which are completely unrealistic.

In this new introduced ideal V,p,T-cycle the heat is partially released isochorically on 2 - 3v, isobarically on 3v - 3p and isothermally on 3p - 3 change of states (see Figure 3). The amounts of heat released are determined with respect to meeting the targets for maximum pressure pmax and temperature Tmax during the cycle. The compression 1 - 2, expansion 3 - 4, emptying 4 - 5 and 5 - 6 as well as the filling 6 - 7 and 7 - 1 are adiabatic. In this ideal cycle no other losses are taken into consideration (i.e. all the processes in this cycle are reversible). During expansion 5 - 6 and filling 7 - 1 the pressure and temperature remain constant.



The theoretical background of the V,p,T-cycle is presented in the Appendix.



Figure 4. Pressure (logarithmic) – Volume Diagram of V,p,T–Seiliger and –Atkinson Cycles

A V,p,T-Seiliger cycle and a V,p,T-Atkinson cycle are simulated and compared, with the help of these analysis instruments. In the following sections, the results of these simulations are presented and discussed.

To facilitate this comparison, the following parameters are kept identical in both cycles: expansion ratio, specific heat q_{zu} (released heat per fluid mass), air-fuel-ratio λ , isentropic exponent κ (constant), maximal pressure p_{max} and temperature T_{max} on the cycle and charge temperature (of the fresh air after compressor and cooler) T₁ (see parameter boxes on the figure).

In the Seiliger cycle the expansion and compression ratios are identical. In the Atkinson cycle, we have chosen a very low compression ratio and a very high charge pressure p_1 , so that the state 1 of the Atkinson cycle is overlaid on the compression curve 1-2 of the Seiliger cycle (see Figure 3). In this way the full potential of the turbo charging can be used without exceeding the maximum pressure (here pmax = 180 bar) and temperature (here Tmax = 2050 K).

The charge pressure p₁ in the Atkinson cycle from Figure 3 is unusually high. Such turbo charging systems are not typical at this time for ICE because the maximum pressure pmax on the cycle limits strongly the level of charge pressure in classic (i.e. Seiliger cycle) applications. For this reason, the current classic, highly supercharged, diesel engines must decrease sharply either the volumetric compression ratio or the aspirated air mass (classical Atkinson and Miller cycle, see e.g. [8]) in order to avoid exceeding the maximum pressure during the cycle. These restrictive measures limit substantially the thermal conversion efficiency of these cycles.

For these reasons we have searched in this paper for a way to make better use of the enthalpy of the exhaust gases. This enthalpy is more than enough to provide the compression of the fresh charge up to the very high pressure p1 of the V,p,T-Atkinson cycle from Figure 3. In this manner no piston work for compressing the fresh charge up to p1 is necessary. On the other hand, the temperature of the fresh charge T1 must be kept low by means of intensive cooling after the compressor stages. The high level of p1, the low level of T1 and the reduced piston work for compression increase considerably the efficiency of thermal conversion □th during this cycle.

In addition the piston work for gas exchange processes becomes strongly positive, i.e. this piston work is supplied for the Atkinson cycle instead of being consumed as in the case of Seiliger cycle (see Figure 4).

As a result, the thermal conversion efficiency of the Atkinson cycle is more than 25% better than that of the Seiliger cycle. At the same time the indicated mean pressure (p_i or imep) of the Atkinson cycle exceeds by more than 70% that of the Seiliger cycle (s. Figures 3 and 4), while meeting the same mechanical and thermal limits in both cycles.

How is that possible?

The states depicted in Figure 5 show that the temperature on the end of compression T2 in the Atkinson cycle is much lower because of the lower volumetric compression ratio of this cycle. Consequently, the isochoric specific heat fraction ψ (needed to achieve pmax) released in Atkinson cycle must be higher than that of the Seiliger cycle. This means that the isothermal specific heat fraction θ released during the Atkinson cycle becomes much lower than that of the Seiliger cycle (see the position of State 3 for the Seiliger cycle in all figures). A bigger isothermal (compared with isochoric and isobaric) specific heat fraction leads to lower thermal conversion

efficiency η_{th} . These facts explain for the most part the better thermal conversion efficiency of the Atkinson cycle.

The diagram of the Figure 6 is presented for a better understanding of the filling and the emptying processes. Along the horizontal curves, the cylinder is closed, between the states 6 - 7 - 1 of the filling process and between the states 4 - 5 - 6 of the empting process. Because in both cycles the temperature at the end of the filling is kept at the same level, the sucked fresh charge mass is bigger in the Atkinson cycle than in the Seiliger cycle. That explains the bigger indicated mean pressure pi of the Atkinson cycle.



Figure 5. Temperature – Volume Diagram of V,p,T–Seiliger and –Atkinson Cycles



Figure 6. Fresh Charge Mass – Volume Diagram of V,p,T–Seiliger and –Atkinson Cycles

The question arises if the well-known temperature – specific entropy diagram (i.e. the T,s – diagram) from Figure 7 can explain the differences between the thermal conversion efficiency of these two cycles.

In the usual ideal ICE cycles – such as the classic Otto, Diesel and Seiliger cycles (see e.g. [1], [2], [3]) - the cylinder is treated thermodynamically as a closed system. In the case of the above presented modified V,p,T-Seiliger and -Atkinson cycles, because the cylinder does not remain closed throughout cycle, i.e. because the gas exchange processes are taken into consideration here, the T,s - diagram does not explain clearly enough all the differences in thermal conversion efficiency between these two cycles. The fact that the specific heat released in both cycles explained above is kept identical means the surface areas under the 2 - 3v - 3p - 3 curves up to abscise (i.e. up to 0 K) are equal in both cycles. In this case, in order to compare

the thermal conversion efficiency of both cycles, it is enough to compare the surface areas under the curves 4 - 5 - 6 - 7 - 1 (i.e. during the gas exchange processes), which represent the discharged heat in T,s – diagram. But during the gas exchange processes, it is assumed above that no heat is discharged (i.e. they are adiabatic).



Figure 7. Temperature – Specific Entropy Diagram of V,p,T–Seiliger and –Atkinson Cycles

Figure 8. Temperature –Entropy Diagram of V,p,T–Seiliger and –Atkinson Cycles

For these reasons, the temperature – entropy (gas mass multiplied by its specific entropy) diagram (i.e. T,S – diagram) from Figure 8 is considered to belong here. The entropy variation of the cycle can be produced not only by means of heat but also as a consequence of the enthalpy H (gas mass multiplied by its specific enthalpy h) exchanges with the manifolds during the gas exchange processes.

It can be easily demonstrated that the surface area under the curves in the T,S – diagram during the adiabatic empting and filling processes is proportional to the enthalpy of the substituted charge (e.g. with the energy lost as a consequence of gas exhaustion).

The ratio of the surface areas under the curves 4-5-6-7-1 and 2-3v-3p-3 up to abscise in both cycles should be considered as a measure of the thermal conversion efficiency in the T,S – diagram. It can be clearly observed in Figure 8 that this ratio is in favor of the Atkinson cycle as a consequence of the lower enthalpy of the exhaust gas which leaves the cylinder.

Please note: The surface areas proportional to the released heat of the cycles in the T,S – diagram are unequal in both cycles (contrary to the T,s – diagram case) because of the different mass of sucked fresh charge on these cycles.

Validation of the ideal V,p,T – Model by use of the Simulation Tool BOOST

The purpose of the Boost simulations is not to obtain a perfect overlapping of the curves in the following diagrams, but rather to demonstrate that the presented V,p,T-model is able to produce good results and accurate predictions of the influence of many parameters upon the thermal conversion efficiency without a big computing effort .



Figure 9. A simple Boost model of a supercharged 6-cylinder diesel engine

The heat release function of the Boost model from Figure 9 was not optimized for overlapping of the cycles. This function is modeled with the help of a simple Vibe function. As a consequence, the pressure peak of the Boost simulation (see Figure 10) exceeds the proposed maximum pressure pmax and has different behaviors depending on temperature and the variation of cylinder volume (see Figures 10 and 11).

Furthermore the valve timing and the valve geometry and lift curves are not optimized for such a high charge pressure p_c . The charge pressure in the block SB1 and the pressure before the turbine in the block SB2 are kept constant and identical with p_1 and p_5 in the ideal V,p,T-model. The heat transfer is deactivated in both models (i.e. Boost- and ideal V,p,T-simulations) of the Atkinson cycle.

The considerable differences between the two simulations can be clearly observed in the high pressure part of Figures 10 respective 12 and in the gas exchange part of Figure 11.

The gas mass variation along the cycle looks almost similar in both simulations. In the Boost simulation the gas mass increases during the high pressure part of the cycle (i.e. between the closing of the intake "ic" and the opening of the exhaust "eo") because of the injected fuel mass. In the V,p,T-model, the gas mass enhancement is neglected (see Figure 13).

Please note: The simulation results from Boost are post processed with the Simulation tool Matlab® (from MathWorks Inc).



Figure 10. Pressure – Volume Diagram of V,p,T– and Boost Models for the Atkinson Cycle



Comparison of Ideal and by Use of Boost (dashed) Simulated Atkinson Cyc

AFR and Charge Pressure and Temperature in both Cycles are identic

Figure 11. Pressure (logarithmic) – Volume Diagram of V,p,T– and Boost Models for the Atkinson Cycle

3. CONCLUSION

The use of Atkinson cycles for enhancing the thermal conversion efficiency of aspirated [7] and turbocharged [8] engines is only successful when the volumetric compression ratio is set lower than the volumetric expansion ratio. In this case a modified crank shaft must be introduced (see Figure 2 and [7]).

In the case of aspirated engines, that leads to a lower indicated mean pressure (imep) during the cycle [4], [7]. For this reason an engine with a bigger displaced volume becomes necessary for achieving the same power as a conventional one.

That disadvantage is negated by supercharged engines because of the possibility to increase the charge pressure while keeping the charge temperature down. Thus it becomes possible to achieve simultaneous a higher imep and a higher thermal conversion efficiency as in the case of the classic supercharged engines.

This behavior is demonstrated in the present paper. The above-presented simple ideal V,p,T-model can describe with accurately enough the complete (i.e. with gas exchange) engine cycle. A formula for the thermal conversion efficiency was developed for this ideal cycle which shows clearly its dependence on many functional parameters (see Appendix).





Figure 12. Temperature – Volume Diagram of V,p,T– and Boost Models for the Atkinson Cycle

Figure 13. Mass – Volume Diagram of V,p,T– and Boost Models for the Atkinson Cycle

Some examples of the way these functional parameter exert their influence are presented in the Appendix, where the volumetric compression ratio ϵ_c of the modified V,p,T-Atkinson cycle is kept constant.

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5. Definitions, Acronyms, Abbreviations and Appendix

Symbol	Meaning	Units
$\varepsilon_{c} = \frac{V_{1}}{V_{2}}$	volumetric compression ration	-
$\epsilon_e = \frac{V_5}{V_2}$	volumetric expansion ratio	-
$p_{max} = p_{3v} = p_{3p}$	maximal pressure on cycle	Pa
$T_{max} = T_{3p} = T_3$	maximal temperature on cycle	К
$V_{max} = V_4 = V_5$	maximal cylinder volume	m ³
$p_C = p_1$	charge pressure after cooler	Pa
$T_C = T_1$	charge temperature after cooler	К
$p_T = p_5$	pressure before turbine	Pa
m ₁	cylinder gas mass in state 1	kg
$m_a = m_1 \cdot \frac{\epsilon_c - 1}{\epsilon_c}$	fresh charge mass per cycle	kg
m _f	fuel mass per cycle	kg
$\kappa = \frac{c^{\circ}_{p}}{c^{\circ}_{v}}$	isentropic exponent	-
c° _p ,c° _v	isobaric & isochoric specific heat capacity	_J kg⋅K
λ	air-fuel ratio (AFR)	-
L _{st}	stoichiometric air requirement ratio	kg≀air kg≀fuel
H _u	fuel lower heating value	<u>J</u> kg
$q_{zu} = \frac{\eta_b \cdot m_f \cdot H_u}{m_1}$	released heat per unit fluid mass	<u>J</u> kg
η _b	released fuel energy completeness	-
$\delta = \frac{q_{zu}}{c_{v'}^{\circ}T_1}$	relative released heat as measure of engine load	-
q _{zu.v}	isochoric part of q_{zu}	J kg
$\psi = \frac{q_{zuv}}{q_{zuv}}$	isochoric released heat ratio	-
qzu.t	isothermal part of q _{zu}	<u>J</u> kg
$\theta = \frac{q_{zu,t}}{q_{zu}}$	isothermal released heat ratio	-
$1-\psi-\theta$	isobaric released heat ratio	-
$\phi = \frac{p_{T}}{p_{C}}$	turbine to compressor pressure ratio	-
$\eta_{th} = \frac{-w_{cycle}}{q_{zu}}$	thermal conversion efficiency	-
Wcycle	specific work on the all cycle	<u>J</u> kg
pi	indicated mean pressure (imep)	bar
η_{sC}, η_{sT}	sentropic efficiency of compressor and turbine	-
io,ic eo,ec	intake valve open & close locations exhaust valve open & close locations	

Formula for the ideal V,p,T-model

$$\begin{split} \delta &= \frac{H_{U} \cdot \eta_{b}}{\lambda \cdot L_{st} \cdot c^{\circ} v \cdot T_{C}} \cdot \frac{\epsilon_{c} - 1}{\epsilon_{c}} \\ p_{max} &= p_{1} \cdot \epsilon_{c} \cdot \left(\epsilon_{c}^{\kappa - 1} + \psi \cdot \delta\right) \\ T_{max} &= T_{1} \cdot \left[\epsilon_{c}^{\kappa - 1} + \psi \cdot \delta + \frac{(1 - \psi - \theta) \cdot \delta}{\kappa}\right] \\ \psi &= \frac{1}{\delta} \cdot \left(\frac{p_{max}}{p_{C} \cdot \epsilon_{c}} - \epsilon_{c}^{\kappa - 1}\right) \\ \theta &= 1 - \psi - \frac{\kappa}{\delta} \cdot \left(\frac{T_{max}}{T_{C}} - \epsilon_{c}^{\kappa - 1} - \psi \cdot \delta\right) \\ a &= \frac{T_{max}}{T_{C} \cdot \delta} = \frac{\epsilon_{c}^{\kappa - 1}}{\delta} + \psi + \frac{1 - \psi - \theta}{\kappa} \\ b &= 1 + \frac{(1 - \psi - \theta) \cdot \delta}{\kappa \cdot \left(\epsilon_{c}^{\kappa - 1} + \psi \cdot \delta\right)} \\ \eta_{th} &= \frac{1}{\delta} \cdot \left[1 - \epsilon_{c}^{\kappa - 1} + \frac{\kappa - 1}{\kappa} \cdot (1 - \psi) \cdot \delta + \frac{\theta \cdot \delta}{\kappa} \dots \right] \dots \\ &+ \frac{\kappa - 1}{\epsilon_{c}} \cdot \left[\epsilon_{c} - 1 - \phi \cdot \left(\epsilon_{e} - 1\right)\right] \\ &+ a \cdot \left[1 - \left(\frac{b}{\epsilon_{e}}\right)^{\kappa - 1} \cdot exp\left(\frac{\theta}{a}\right)\right] \end{split}$$

Requirements for cycle realization

1. Requirement (for maximal pressure)

$$\begin{split} p_{max} &\geq p_{C} \cdot \epsilon_{c}^{-\kappa} \quad \text{i.e.} \qquad \psi \geq 0 \\ p_{max} &\leq p_{C} \cdot \epsilon_{c} \cdot \left(\epsilon_{c}^{-\kappa-1} + \delta \right) \end{split}$$

2. Requirement (for maximal temperature)

$$\begin{split} & \mathsf{T}_{max} \geq \mathsf{T}_{C} {\cdot} \epsilon_{c}^{\kappa-1} \\ & \mathsf{T}_{max} \leq \mathsf{T}_{C} {\cdot} \left(\epsilon_{c}^{\kappa-1} + \psi {\cdot} \delta + \frac{\delta}{\kappa} \right) \end{split}$$

3. Requirement (for heat release)

$$1 - \psi - \theta \ge 0$$
 and $\theta \ge 0$

4. Requirement (for turbocharging)

$$\eta_{sT} \cdot \left(m_a + m_f\right) \cdot c^\circ_p \cdot T_T \cdot \left(1 - \frac{T_u}{T_T}\right) > \frac{m_a \cdot c^\circ_p \cdot T_u}{\eta_{sC}} \cdot \left[\left(\frac{p_C}{p_u}\right)^{\kappa} - 1 \right]$$

where index u denotes the ambient state

Influence upon η_{th} of modified Atkinson cycle ($\epsilon_c = 5$)



